# Next-Generation Global and Mesoscale Atmospheric Models

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## LONG-TERM GOALS

The long-term goal of this research is to construct global and mesoscale nonhydrostatic numerical weather prediction (NWP) models for the U.S. Navy using new numerical methods specifically designed for modern computer architectures. To take full advantage of distributed-memory computers, the global domains of these new models are partitioned into local sub-domains, or elements, that can then be solved independently on multiple processors. The numerical methods used on these sub-domains are local, high-order accurate, fully conservative, and highly efficient. Using these ideas we are developing global and mesoscale nonhydrostatic atmospheric models that will improve upon the operational models currently used by all U.S. agencies including the U.S. Navy.

## OBJECTIVES

The objective of this project is to construct new high-order local methods for the Navy's next-generation global and mesoscale nonhydrostatic NWP models. The high-order accuracy of these methods will ensure that the new model yields better forecasts than the current global spherical harmonics model (NOGAPS) and better accuracy than the current mesoscale finite difference model (COAMPS). The objective is to achieve this accuracy while increasing the geometric flexibility to use any grid as well as to increase the efficiency of these models on large processor-count distributed-memory computers. Higher efficiency means that the new models will require less computing time that then allows for increasing the number of ensemble members and/or increasing the resolutions of the NWP models. The methods that we propose to use for these models are state-of-the-art and are not being used by either current or newly emerging NWP and climate models.

## APPROACH

To meet our objectives we explore:

1. spectral element (SE), discontinuous Galerkin (DG), and WENO spatial discretization methods;
2. high-order semi-implicit (SI) time-integrators with adaptive time-stepping for vastly improved efficiency;

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**14. ABSTRACT**
**The long-term goal of this research is to construct global and mesoscale nonhydrostatic numerical weather prediction (NWP) models for the U.S. Navy using new numerical methods specifically designed for modern computer architectures. To take full advantage of distributed-memory computers, the global domains of these new models are partitioned into local sub-domains, or elements, that can then be solved independently on multiple processors. The numerical methods used on these sub-domains are local, high-order accurate, fully conservative, and highly efficient. Using these ideas we are developing global and mesoscale nonhydrostatic atmospheric models that will improve upon the operational models currently used by all U.S. agencies including the U.S. Navy.**

**15. SUBJECT TERMS**

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3. high-order Lagrangian-like time-integrators that are fully conserving and scale well on modern computer architectures;

4. various forms of the governing equations in order to maximize accuracy, efficiency, stability, and conservation properties;

5. unified hydrostatic, nonhydrostatic, and pseudo-incompressible formulations of the equations; and

6. fully unstructured and adaptive grids.

The power of SE and DG methods is that they are high-order accurate yet are completely local in nature – meaning that the equations are solved independently within each individual element and processor. Furthermore, high-order methods have minimal dispersion error – this is an important property for capturing fine-scale atmospheric phenomena (e.g., tropical cyclones, Kelvin and Rossby waves). The theoretical development of SE and DG methods are now well-established and these methods are currently the two most successful methods found in the literature for fluid flow problems, with WENO also being quite popular for structured grid applications.

Semi-implicit (SI) and Lagrangian time-integrators offer vast improvements in efficiency due to the longer time steps that they permit; it should be mentioned that semi-implicit and Lagrangian-like methods can be classified together under the heading of implicit-explicit (IMEX) methods that has garnered much attention in the computational mathematics literature. Furthermore, in order to reap the full benefits of the high-order spatial discretization methods requires increasing the order of accuracy of the time-integration methods as well; this is a topic that too often has been ignored by most scientific computing communities, including the NWP community. Lagrangian methods have not been used successfully for mesoscale modeling because of their lack of conservation. Another problem that they pose is that they require vast amounts of inter-processor communication on a distributed-memory computer. We have worked on Lagrangian-like methods that are conserving and require no additional inter-processor communication.

Before committing resources towards the development of new NWP models, it is important to identify the form of the governing equations that is most capable of conserving all quantities deemed important. We have been performing studies on this topic for the past two years – that is, to identify the form of the governing equations capable of representing conservation of either mass, energy, or both. In addition, we have analyzed various forms of the governing equations with respect to robustness, flexibility, and efficiency in the context of implicit-explicit (IMEX) time-integration methods. Within this work we will also explore hybrid models that solve either the hydrostatic, nonhydrostatic, or pseudo-incompressible equations. This feature allows the models to be used for research purposes by Navy scientists in order to test the importance of multi-scale phenomena at specific resolutions.

One final area that needs to be explored is the concept of adaptive grids. In the past few years, adaptive grids have gained considerable momentum in the atmospheric modeling community – in fact, I was invited to give a keynote lecture at the University of Reading in March 2009 to kick-off a year long program on adaptive modeling at the Newton Institute in Cambridge University, England.

## WORK COMPLETED

In this section, we describe the work completed this fiscal year. The work can be categorized into four sections: analysis of the best form for the nonhydrostatic equations, time-integration methods for these equations, spatial discretization methods, and physical parameterization.

*Nonhydrostatic Governing Equations.* We have completed our analysis of the various forms of the Euler equations and their advantages/disadvantages for nonhydrostatic modeling. Specifically we analyzed the following equations: Set 1 is defined as follows where the solution vector is Exner

$$\frac{\partial \pi}{\partial t} + \mathbf{u} \bullet \nabla \pi + (\gamma - 1)\pi(\nabla \bullet \mathbf{u}) = 0; \quad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \bullet \nabla \mathbf{u} + c_p \theta \nabla \pi = -f(\mathbf{k} \times \mathbf{u}) - g\mathbf{k}; \quad \frac{\partial \theta}{\partial t} + \mathbf{u} \bullet \nabla \theta = 0$$

pressure, velocity, and potential temperature. Set 1 is the equation set used in the U.S. Navy's mesoscale model COAMPS. The main problem with this equation set is that it cannot conserver either mass or energy.

Set 2 in conservation form (denoted as Set 2C) is defined as follows where the solution vector is

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \mathbf{U} = 0; \quad \frac{\partial \mathbf{U}}{\partial t} + \nabla \bullet \left( \frac{\mathbf{U} \otimes \mathbf{U}}{\rho} + P\mathbf{I}_2 \right) = -f(\mathbf{k} \times \mathbf{U}) - \rho g\mathbf{k}; \quad \frac{\partial \Theta}{\partial t} + \nabla \bullet \left( \frac{\Theta \mathbf{U}}{\rho} \right) = 0; \quad P = P_A \left( \frac{R\Theta}{P_A} \right)^\gamma$$

density, momentum, and density potential temperature. Set 2 is the form used in WRF. This form is very attractive because it conserves mass although it does not conserve energy. It does, however, conserve density potential temperature which is related to entropy.

This set is of interest because it can also be written in non-conservation form while still conserving mass. We shall refer to it as Set 2NC (for non-conserving). and it is defined as follows:

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \bullet \nabla \rho + \rho \nabla \bullet \mathbf{u} = 0; \quad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \bullet \nabla \mathbf{u} + \frac{1}{\rho} \nabla P = -f(\mathbf{k} \times \mathbf{u}) - g\mathbf{k}; \quad \frac{\partial \theta}{\partial t} + \mathbf{u} \bullet \nabla \theta = 0; \quad P = P_A \left( \frac{\rho R\theta}{P_A} \right)^\gamma$$

The interest in equation Set 2NC is that it conserves mass and offers much flexibility in the type of time-integrators that can be used with it. For example, note that the first two terms (in red font) of each of the components of mass, momentum, and potential temperature can be recast as a Lagrangian derivative. This then allows the use of Lagrangian-like time-integrators. Set 3 is defined as:

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \mathbf{U} = 0; \quad \frac{\partial \mathbf{U}}{\partial t} + \nabla \bullet \left( \frac{\mathbf{U} \otimes \mathbf{U}}{\rho} + P\mathbf{I}_2 \right) = -f(\mathbf{k} \times \mathbf{U}) - \rho g\mathbf{k}; \quad \frac{\partial E}{\partial t} + \nabla \bullet \left( \frac{E + P}{\rho} \mathbf{U} \right) = 0; \quad P = (\gamma - 1)\left( E - \frac{\mathbf{U} \bullet \mathbf{U}}{2\rho} - \rho\phi \right)$$

where the solution vector is density, momentum, and density total energy, where E=$\rho$e with e=$c_v$T +0.5$\mathbf{u} \bullet \mathbf{u}$ + $\phi$, in other words e represents internal, kinetic, and potential energy. This equation set is not used in atmospheric modeling but is the equation of choice in computational fluid dynamics (CFD). This set is very attractive because it conserves both mass and energy regardless of whether the flow is inviscid or viscous (with the proper viscous stressed included) (see Ref. [3]). One question we had

about this set, however, is whether it could be coupled to existing physical parameterization packages that rely on potential temperature and not energy; this question has been answered in the past year with the recent results of the Japanese NICAM model which in fact uses this equation set.

The final equation set studied is Set 4 that is written as follows:

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \bullet \nabla \rho + \rho \nabla \bullet \mathbf{u} = 0; \quad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \bullet \nabla \mathbf{u} + \frac{1}{\rho} \nabla P = -f\left(\mathbf{k} \times \mathbf{u}\right) - g\mathbf{k}; \quad \frac{\partial P}{\partial t} + \mathbf{u} \bullet \nabla P + \gamma P \nabla \bullet \mathbf{u} = 0$$

Note that this equation is also amenable to various time-integration methods including Lagrangian methods since the first two terms of each of the components of the equations can be written as a Lagrangian derivative. Furthermore, as in Sets 2C and 2NC, Set 4 can formally conserve mass but not energy. Set 4 is a very good compromise between conservation and efficiency. Note that Sets 1 and 4 are the only fully closed systems requiring no equation of state, whereas Sets 2C, 2NC, and 3 all require an equation of state in order to couple the extra variable (pressure) to the prognostic variables. Set 4 is the equation set used by the LM model of the German Weather Service and has some good properties that we are analyzing.

Using these 5 equation sets we developed x-z slice mesoscale models to: compare the spectral element and discontinuous Galerkin methods, analyze semi-implicit time-integrators, and to see how these models behaved under a series of test cases including sharp front simulations and nonhydrostatic flow over mountains. This work resulted in a peer-reviewed article that appeared this year (see Ref. [3]) and another that has been submitted (see Ref. [6]). The summary of those papers is that set 2NC is the form that we should use in order to construct the optimal nonhydrostatic model, taking into account: accuracy, efficiency, and conservation measures. We shall next use this equation set to build a 3D global/local nonhydrostatic model.

*Time-Integrators*. Directly connected with our choice of equation sets is the resulting semi-implicit operators. The specific equation set chosen contributes to the overall efficiency of the model (e.g., set 1 and 4 are inherently faster than the other sets since they do not require an equation of state) but the semi-implicit formulations also have a strong effect on the efficiency of the models. We found that sets 1 and 4 are indeed the fastest codes but the fact that they do not conserver either mass or energy makes them unattractive. On the other hand, set 2NC was the third fastest code while conserving mass and energy very well. In studies performed this year, we compared semi-implicit time-integrators both in their Schur (i.e., pseudo-Helmholtz) and No Schur (i.e., full system) forms and compared them to the types of explicit time-integrators currently being used in split-explicit models. Our results show that if the Schur form is used, then the semi-implicit models are always faster than explicit models.

*Spatial Discretization Methods*. We have been arguing in the course of this three year project that the best next-generation models will be those based on element-based Galerkin (EBG) such as the spectral element and discontinuous Galerkin methods. However, we have only partly showed the benefits of this approach such as: high parallel efficiency and high-order accuracy. We have also hinted in the past that another attribute of these methods is that they allow one to use unstructured and adaptive grids. We have teamed up with the University of Mainz (Mainz Germany) and the University of Hamburg (Hamburg Germany) to combine the triangle-based discontinuous Galerkin method with adaptivity. Although only preliminary, the results show that indeed this combination will be quite formidable for tackling nonhydrostatic problems (such as tracking hurricanes, etc.).

We are always looking in the literature for new and promising methods to use for our models. The weighted essentially non-oscillatory (WENO) method has received much attention in computational fluid dynamics and so we have begun exploring it for the construction of nonhydrostatic atmospheric models. So far, we have implemented the method on a two-dimensional tracer problem, and a two-dimensional Euler code but without gravity. The addition of gravity severely complicates the solution strategy for this method but we hope to show results for a full nonhydrostatic WENO model in the next few months.

*Physical Parameterization.* In collaboration with Sasa Gabersek and Jim Doyle of the Naval Research Laboratory in Monterey, CA, we have included simple microphysics to the semi-implicit spectral element Euler codes. We have combined the expertise of NPS on dynamical cores and numerical methods with the expertise of NRL-Monterey on physical parameterizations and have implemented a simple microphysics known as Kessler physics. The idea is to include three tracer equations into the governing equations. These three tracers represent: water vapor, condensation, and precipitation. To include the microphysics requires modifying the equation of state such that density now includes both dry and moist air, and the potential temperature becomes virtual potential temperature. Our results show that the model is perfectly stable for very long time-integrations and that the results produced by the model are quite physical. This work has been summarized in a journal article soon to be submitted (in October 2009, see Ref. [8]).

## RESULTS

*Time-Integration Methods.* We explored various time-integration strategies in order to increase the efficiency of the nonhydrostatic models. In Ref. [6] we analyze explicit time-integration strategies (using 3$^{rd}$ Runge-Kutta methods as in the split-explicit approach) with semi-implicit time-integrators based on backward difference formulas (BDF2).



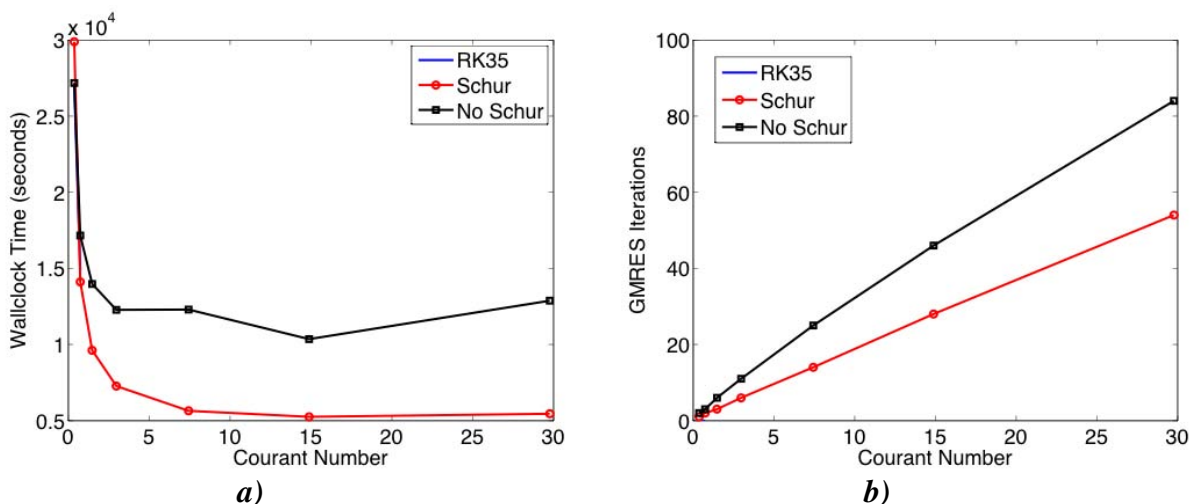*a)*                                                                 *b)*

*Figure 1: Performance study of the explicit (RK35) and semi-implicit (Schur and No Schur) models for the warm bubble test for various Courant numbers  a) wallclock time and b) number of GMRES iterations.*

In Figure 1 we show that while the explicit RK3 method requires prohibitively small Courant numbers (time-step size), the semi-implicit methods in either their Schur or No Schur form can use Courant numbers far greater than one (up to 30). Figure 1a (left panel) shows the wallclock time of the explicit (RK35), semi-implicit with Schur complement (Schur), and semi-implicit for the full system (No Schur). The semi-implicit methods outperform the explicit methods. Figure 1b (right panel) shows the average number of GMRES iterations per time-step; GMRES is the iterative solver used to solve the linear system. Clearly, the advantage of the Schur form extends beyond just solving a smaller problem – the Schur form is far better conditioned thereby requiring fewer iterations.

The results of this study are sufficiently encouraging to convince us to extend the models to three-dimensions. However, one other possibility that we are exploring involves solving the equations fully-implicitly. This means that the equations have to be solved using a Newton outer iteration and a GMRES inner iteration. We are developing a prototype using this idea and will report our findings in the near future. For now, we can show that the fully-implicit codes are still not as efficient as our semi-implicit codes as is clearly illustrated in Figure 2 where the number of GMRES iterations as a function of time (time step refers to the increasing time level) are plotted for the Jacobian-free Newton-Krylov (fully-implicit) and No Schur semi-implicit methods. The advantage of the fully-implicit method, however, is that it can use as large a time-step as desired without fear of numerical instabilities. We are presenting these results at the AGU meeting in San Francisco.
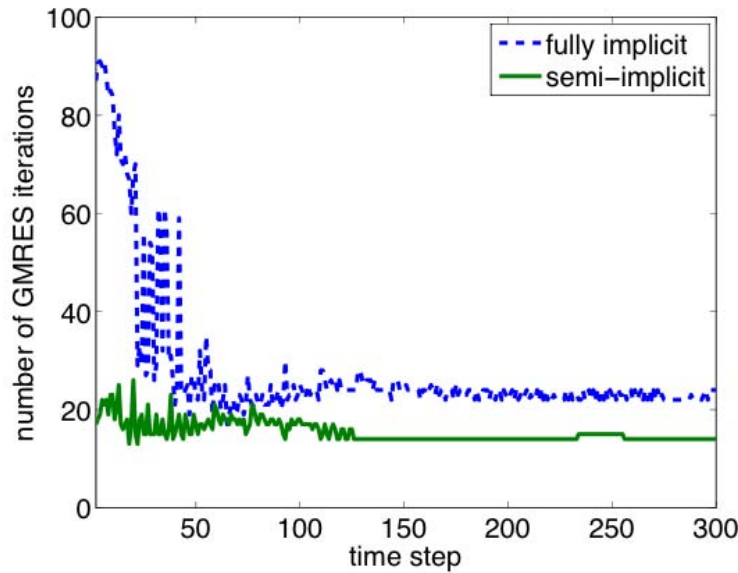


***Figure 2: Performance study of the fully-implicit and semi-implicit (No Schur) models for the warm bubble test.***

*Spatial Discretization Methods.* In all of our previous work we have discretized the equations in space using element-based Galerkin methods that have been touted as being geometrically flexible meaning that unstructured adaptive grids can be used. All of our models, in fact, have used unstructured grid machinery (e.g., NSEAM) but these grids are comprised of quadrilateral elements. Quadrilateral elements can only be used to construct efficient adaptive models in a non-conforming approach (meaning that the grid will have "hanging" nodes). We are only beginning to study this approach but

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have much experience in conforming adaptive methods based on triangles. In collaboration with the University of Mainz and the University of Hamburg, we are coupling the triangle-based discontinuous Galerkin (DG) methods developed at NPS with the adaptive mesh refinement developed at Hamburg. Figure 3 shows the product of this collaboration where simulations of warm bubbles, inertia gravity waves, and density current simulations have been conducted using triangle-based DG methods with grid adaptivity. Clearly, only small overshoots can be seen even though this problem represents a discontinuity in potential temperature. The reason why the model remains stable is due to the ability of the adaptive grid to capture the regions of steep gradients.
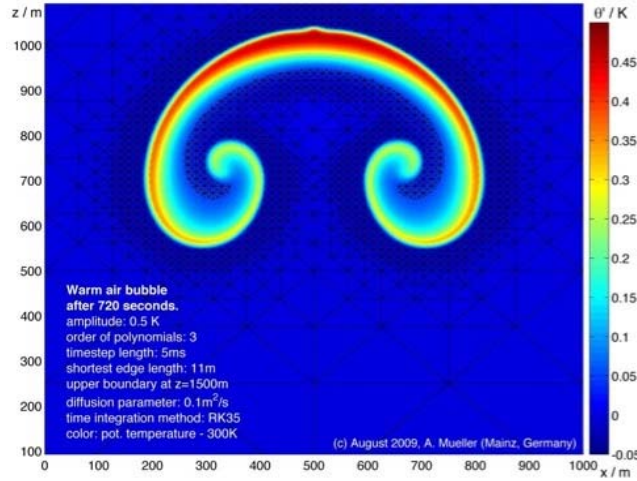


***Figure 3: Potential Temperature contours for a warm bubble test after 720 s for the semi-implicit DG model with a Courant number of 50 and adaptive triangular grids.***

Although these results are extremely encouraging, we continue to search for better methods or perhaps ways of combining DG with other methods. A good example is the weighted essentially non-oscillatory (WENO) method. This method is a high-order finite difference method constructed specifically for resolving discontinuities. We envision using WENO either as a stand-alone Euler solver, only for the tracers, or as a way to handle the overshoots and undershoots in both spectral element and discontinuous Galerkin methods.



a)                                           b)                                           c)
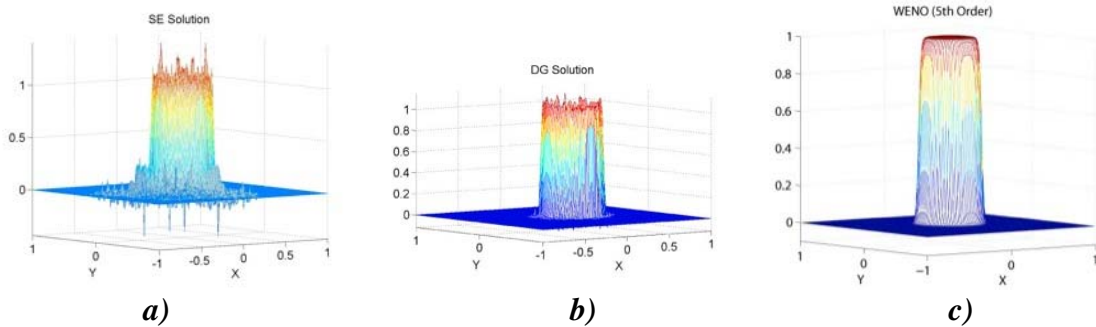
***Figure 4: Solution after one revolution for the 2D advection equation for a) SE, b) DG, and c) WENO. Note, however, that WENO is run with a limiter while SE and DG are not.***

We have begun our study of WENO by first comparing it to the spectral element (SE) and discontinuous Galerkin (DG) methods for the two-dimensional advection equation (i.e., a tracer equation). Figure 4 shows the solution after one revolution of a cylindrical initial condition. Figure 4a) shows the solution for the SE method, Fig. 4b) for the DG method and Fig. 4c) for the WENO method; keep in mind that neither the SE nor the DG methods use a limiter to handle oscillations whereas the WENO method does. We only show this result to motivate the need for including WENO in our studies. We anticipate using WENO as a possible limiter for our SE and DG models in order to remove oscillations from the solutions.

We are also considering WENO as a possible candidate for a next-generation model but a few hurdles have to be overcome. We currently have constructed an Euler model using WENO but only without gravity. The next challenge that remains is to include gravity but currently no such formulation exists and it is not so straightforward to extend WENO to non-homogeneous equations.

*Physical Parameterization.* In order to better test our nonhydrostatic models, it is important not only to run challenging dry physics tests but also those including moist physics. The simplest moist physics test that can be implemented is the Kessler physics problem. With this microphysics, three additional tracer equations are added to the Euler equations. These three equations represent water vapor, condensation (i.e., clouds), and precipitation (i.e., rain). In collaboration with scientists in the mesoscale modeling group of the Naval Research Laboratory, we have coupled the Euler solver with the Kessler microphysics. Figure 5 shows a snapshot after 1800 seconds for the water vapor (color map), condensation (grey lines), and precipitation (white). This snapshot shows that the model remains stable (for all time) and produces realistic squall lines (series of thunderstorms generated in sequence). In fact, to gain a full appreciation of the high-quality of these results requires looking at an animation which can be found on the website listed above.

The group at NRL has conducted a detailed analysis of these results; more on this can be found in the ONR report by Dr. Jim Doyle. The results of this study have been collected and will be submitted shortly to a peer-reviewed journal (see Ref. [8]).
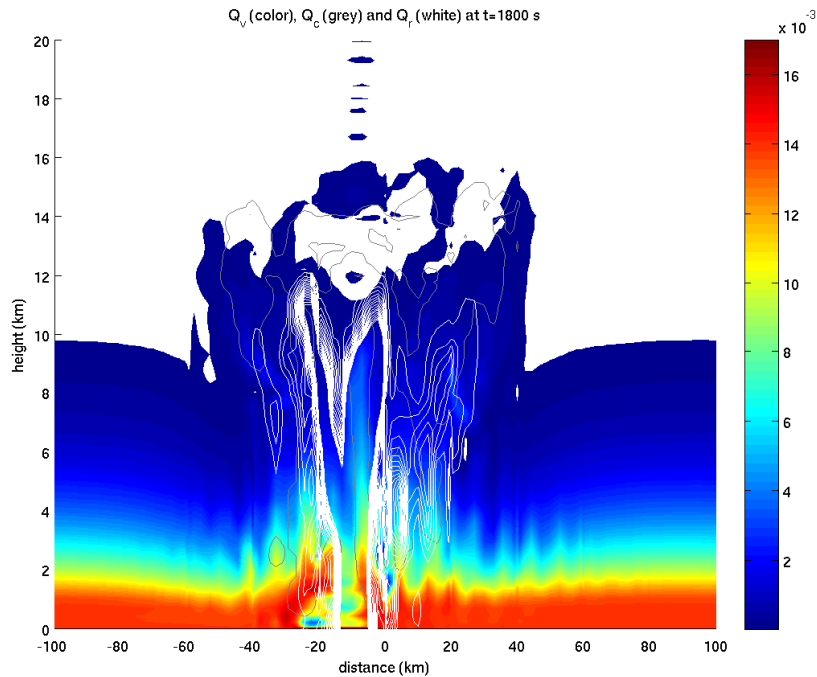
***Figure 5: Concentrations of water vapor (Qv), condensation (Qc), and rain (Qr) for a Squall Line experiment using simple microphysics with the Semi-implicit Spectral Element Nonhydrostatic Model. The results are shown after 1800 seconds showing the generation of condensation (grey lines) and precipitation (white patches inside grey lines).***

## IMPACT/APPLICATIONS

NOGAPS and COAMPS are run operationally by FNMOC and is the heart of the Navy's operational support to nearly all DOD users worldwide. This work targets the next-generation of these systems for massively parallel computer architectures. NSEAM and its mesoscale cousins have been designed specifically for these types of computer architectures while offering more flexibility, robustness, and accuracy than the current operational systems. Additionally, the new models are expected to conserve all quantities such as mass and energy and use state-of-the-art time-integration methods that will greatly improve the capabilities of the Navy's forecast systems.

## TRANSITIONS

Improved algorithms for model processes will be transitioned to 6.4 as they are ready, and will ultimately be transitioned to FNMOC.

## RELATED PROJECTS

Some of the technology developed for this project could be used to improve NOGAPS in other NRL projects. The work performed in this work unit on coupling NSEAM with NOGAPS physics has already revealed some sensitivities of the physical parameterization to the vertical coordinate; this information can now be used to improve the forecasts of NOGAPS. In addition, the work on the

mesoscale models will help improve COAMPS. An example is the time-integration methods that we are exploring for the new models may well be incorporated into the current operational version of COAMPS. The insight gained on grids in the current project could also be leveraged to develop a global version of COAMPS by virtue of the cubed-sphere grid (hexahedral) and data structures developed for the NSEAM model.

## PUBLICATIONS

### Journals

1. Läuter, M., Giraldo F.X., Handorf D., and Dethloff K., 2008: A Discontinuous Galerkin Method for the Shallow Water Equations with Spherical Triangular Coordinates. *Journal of Computational Physics*, Vol. 227 pp. 10226-10243 (2008).

2. Kim, Y.J., Giraldo F.X., Liou C.S., Flatau M., and Peng M., 2008: The Kelvin Wave and the Madden-Julian Oscillation in Aqua-planet Simulations by the Navy's Spectral Element Atmospheric Model. *Journal of Geophysical Research*, Vol. 113, D20102, 16 pp. (2008).

3. Restelli, M., and Giraldo F.X., 2009: A Conservative Semi-Implicit Discontinuous Galerkin Formulation for the Navier-Stokes Equations in Nonhydrostatic Mesoscale Modeling. *SIAM Journal of Scientific Computing*, Vol. 31, pp. 2231-2257 (2009).

4. Dea, J., Giraldo F. X., and Neta B., 2009: High-Order Non-Reflecting Boundary Conditions for the Linearized 2D Euler Equations. *Wave Motion*, Vol. 46, pp. 210-220 (2009).

5. Giraldo, F.X., and Restelli M., 2009: High-Order Semi-Implicit Time-Integrators for Discontinuous Galerkin Methods. *International Journal for Numerical Methods in Fluids*, [in press].

6. Giraldo, F.X., Restelli M., and Läuter, M., 2009: Semi-Implicit Time-Integrators for the Navier-Stokes Equations: Applications to Nonhydrostatic Atmospheric Modeling. *SIAM Journal on Scientific Computing*, [submitted].

7. Läuter, M., Giraldo, F.X., Reich, S., Restelli, M., Handorf, D., and Dethloff, K. 2009: A 2D Unified (Non-) Hydrostatic Model of the Atmosphere with a Discontinuous Galerkin Method. *Journal of Computational Physics* [submitted].

8. Gabersek, S., Giraldo, F.X., and Doyle, J.D., 2009: Simple Microphysics Experiments with a Spectral Element Model. *Monthly Weather Review* [in preparation].

9. Lindquist, J., Neta, B., and Giraldo, F.X., 2009: Spectral Element Solution of the Klein-Gordon Equation with High-Order Boundary Treatment and Time. *Wave Motion* [in press].

### Theses

1. Dea J., 2009: *High-Order Non-Reflecting Boundary Conditions for Linearized Euler Equations*. PhD Thesis, Naval Postgraduate School [published].

2. Gibbons, S.L., 2009: *Impact of Sigma Coordinates on Continuous and Discontinuous Galerkin Navier-Stokes Models for Nonhydrostatic Modeling*. Master's Thesis, Naval Postgraduate School [published].

**Conference Abstracts**

1. Giraldo, F.X., Restelli M., and Läuter, M., 2009: Semi-Implicit Time-Integrators for the Navier-Stokes Equations: Applications to Nonhydrostatic Atmospheric Modeling. *SIAM Conference on Mathematical Issues in the Geosciences*, Leipzig Germany (June 2009).

2. Kelly, J.F., Giraldo, F.X., and Carr, L.E., 2009: Jacobian-free Newton-Krlyov Solvers for a Spectral Element Navier-Stokes Model. *American Geophysical Union Fall Meeting*, San Francisco, CA. (December 2009).

**Invited Talks**

1. Giraldo, F.X., 2009: Implicit Time-Integrators for Adaptive Methods in Atmospheric Modeling. University of Reading, Reading UK (March 2009).

2. Giraldo, F.X., 2009: Continuous and Discontinuous Galerkin Methods for Geophysical Fluid Models. Department of Applied Mathematics, University of Karlsruhe, Karlsruhe Germany (July 2009).

3. Giraldo, F.X., 2009: Why Do We Need More Atmospheric Models. Alfred Wegener Institute, Potsdam Germany (July 2009).